DECISION MODEL FOR VERNIER ACUITY TASKS

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Abstract

Vernier acuity tasks are known for their accuracy of 2 - 4 sec of arc, which corresponds to 1/10 of the size of a receptor. Considering the performance of an ideal detector we show that the attainable accuracy, which is limited by neural noise rather than by receptor size, is somewhat better than the experimental results. In light of these results we propose that in relative localization tasks the visual system may be modelled as a nearly-optimal estimator.

Introduction

The extremely high sensitivity exhibited in Vernier acuity is a well known phenomenon of visual acuity; for example, misalignment thresholds of two abutting line-segments, when one of them is shifted laterally, is about 2 - 4 sec of arc [1,2] in comparison with a threshold of about 30 sec of arc in two points resolution experiments. To appreciate the result one should note that such a displacement threshold corresponds to a length of 10 - 20μ seen at a distance of 1 meter, whereas in terms of retinal dimensions it amounts to about 1/10 of the size of a photoreceptor in the center of the fovea.

Such findings have puzzled many vision researchers because of the misconception and confusion of acuity and resolution. This is the reason why Begbie [3], for example, brought forward the following question: "... if the retinal cone is the smallest retinal unit in the fovea, how can one use the retinal mosaic to determine distances that are only a fraction of this unit?". It is however true that the dimensions of the retinal mosaic and the visual optics impose limitations on the resolving power of the eye, and Westheimer [4] is indeed right in stating that "In the limit no eye can do better than its optics". But in localization tasks, and we consider Vernier acuity as such, the limiting factor seems to be the neural noise rather than the receptor size.

In this paper we consider the performance of an ideal detector and show that the upper bound on the accuracy of Vernier acuity tasks, limited by the noise, is not in contradiction to the experimental results. This suggests that at the perceptual decision level the visual system may be modelled as an estimator and hypothesis testing system based on likelihood function [5].

Our approach has in fact been influenced by the successful application of such a concept in audition [6,7] and color vision [8].

The Model

Our model is based on the following assumptions:

1. The temporal statistics of the signal at the retinal output level is representable as a compound Poisson process [9]. This is inferred from analysis of auditory signals [10,11].

2. For a known visual signal, the Poisson processes corresponding to different receptors are independent.

3. The line spread function of the optical system is Gaussian.

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For the sake of simplicity we do not consider here the effects of inhibition and adaptation therefore dealing with the one-dimensional case of localization of a straight line. However, it is to be noted that since the visual system does not have an absolute reference system, the localization problem is one of relative localization.

Considering assumption (3), the retinal light distribution - due to a straight line stimulus - can be described by:

$$I(x/\alpha) = \frac{I_o}{\sqrt{2\pi}\rho} \exp\left[-\frac{(x-\alpha)^2}{2\rho^2}\right] , \tag{1}$$

where α denotes the line's position, ρ is a parameter describing the line spread function of the optical system, and I_o is the luminance.

It can be shown that the likelihood function is given by [9]:

$$\ell(\alpha) = \ell n p[\{X;\ 0 \leq t \leq T\}/\alpha] = T \sum_i \int_{R_i} I(x/\alpha)dx + \sum_i N_T(R_i) \ell n \int_{R_i} I(x/\alpha)dx , \tag{2}$$

where X denotes the sample of the process received from the receptors, $p[\{X;\ 0 \leq t < T\}/\alpha]$ is the sample function density, T the sampling interval, R_i the i-th receptor, and $N_T(R_i)$ the number of impulses received from the i-th receptor during the sampling interval. It is important to stress that $N_T(R_i)$ is a sufficient statistics for α [9].

The Cramer-Rao lower bound on the variance of the estimator $\hat{\alpha}$ of the unknown parameter α is given by [12].

$$\mathrm{Var}[\hat{\alpha}] \geq \frac{1}{E\left[\frac{\partial \ell(\alpha)}{\partial \alpha} / \alpha\right]^2} , \tag{3}$$

where, in the case considered here, we have

$$E\left[\frac{\partial \ell(\alpha)}{\partial \alpha} / \alpha\right]^2 = \sum_i \int_0^T \left\{\left[\int_{R_i} I(x/\alpha)dx\right]^{-1} \left[\frac{\partial}{\partial \alpha} \int_{R_i} I(x/\alpha)dx\right]^2\right\} dt =$$

$$= \sum_i \frac{kI_oT}{\sqrt{2\pi}\rho} \frac{\int_{R_i} \exp\left[-\frac{(x-\alpha)^2}{2\rho^2}\right] \frac{d(x-\alpha)^2}{2\rho^2}}{\int_{R_i} \exp\left[-\frac{(x-\alpha)^2}{2\rho^2}\right] dx} \tag{4}$$

where k is the firing rate given in impulses per second per luminance unit.

Taking ρ as equal to the size of a receptor [4], one obtains:

$$\sigma_{\hat{\alpha}} = \sqrt{\mathrm{Var}[\hat{\alpha}]} \geq \frac{\rho}{\sqrt{\frac{kI_o}{\sqrt{2\pi}} T}} . \tag{5}$$

Since an efficient estimator of α exists (at least asymptotically), we may consider the bound with equality and obtain for the accuracy of the localization task

$$\sigma_{\hat{\alpha}} = \frac{\rho}{\sqrt{\frac{kI_o}{\sqrt{2\pi}} T}} . \tag{6}$$

From this result it appears as though the performance of the system can improve by increasing the sampling interval so that one can obtain any desired accuracy. However, the sampling interval is shown in many psychophysical experiments to be limited to approximately 1 sec and therefore increasing T over 1 sec does not have any appreciable effect on the performance of the system.

Thus, for $T = 0.8 \div 1.0$ sec [4], and $kI_0/\sqrt{2\pi} = 400 \div 1000$ impulses/sec [13], we obtain

$$\sigma_{\hat{a}} = \frac{\rho}{30} \div \frac{\rho}{10}$$

or $1 \div 3$ sec of arc.

Discussion

We have shown in this paper that the experimental results do not contradict the theoretical bound on the performance of an ideal detector, and in fact our analysis suggests that the system is only suboptimal. We are aware of the fact that there is not enough available data on the statistics of the neural signal in higher levels of the system and think that better bounds can be calculated as more and new data will be available. Experiments are indeed now performed in the visual research laboratory of the Technion - Israel Institute of Technology, in order to learn and model more accurately the statistics of the neural signal in the visual system. At this level it seems to us premature to offer a possible neural model for the realization of a maximum likelihood estimator although one can think of more than one.

To sum up, in this short note we answered the above mentioned question of Begbie [3] and others, and offer an alternative explanation to the way in which the visual system may attain the given accuracy in Vernier acuity tasks in spite of the apparent limitations imposed by the retinal and optical structures.

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