

Fiber-optic reading magnifiers for the visually impaired

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Received January 9, 1995; revised manuscript received April 17, 1995; accepted April 25, 1995

We describe fiber-optic stand magnifiers specifically designed for use as low-vision reading aids. Application of this technology results in better optical and ergonomic properties. The fiber-optic magnifiers (tapers) provide bright, uniformly illuminated, distortion-free images. The reading material can be scanned without the user's having to bend directly over the magnifier. One can further increase the scanning field by slanting the taper to tilt the image toward the observer. Tilting the upper face of the taper by cutting the smaller lower face on a bias is shown to increase the scanning range substantially and to provide better control of illumination. The scanning range of such tilted tapers is approximately double that of the equivalent lens magnifiers. To increase the contrast transfer through the magnifiers, we have developed lower-resolution tapers with an increased core-to-cladding ratio. The increase in contrast transfer is reported for representative tapers. The lower-resolution design is also helpful in reducing the manufacturing cost of taper magnifiers.

1. INTRODUCTION

We describe the application of fiber-optic technology to the development of stand magnifiers of improved optical and ergonomic properties compared with existing devices. Stand magnifiers are popular visual aids frequently prescribed in vision rehabilitation services and are often purchased over the counter by patients or their relatives. Thus, these devices often serve as the only aid for many visually impaired persons. Their main advantages over other reading aids are their stability and their ease of use. The fixed, stable distance from the reading material, and the flexibility in the distance from the eye to the magnifier, make their use easy and simple to learn. The stable imaging conditions eliminate the fluctuation of vision that older patients frequently experience because of poor motor control when they are using high-add spectacles or hand-held magnifiers. Although spectacles are considered superior for extended, continuous reading, many patients prefer stand magnifiers for a variety of daily activities, including spot reading of personal mail, recipes, labels on food boxes, price tags, etc. These activities are associated with independent living and thus are perceived to be even more important than pleasure reading of books or magazines.

Existing stand magnifiers have significant limitations. A major complaint of patients concerns the need for bending over the magnifier to be able to read through it.¹ When using it at a desk, for example, the user has to lean forward and closely follow the position of the magnifier on the page with his or her whole body. This requirement is of little consequence for young patients but is very tiring for elderly users. Reading stands are frequently recommended as a partial solution to this difficulty.²

The postural problems associated with the stand magnifier also interfere with the attainment of proper illumina-

tion of the printed material. The user's head tends to block light from above, whereas the need to permit the access of light through the sides of the magnifier often makes for an uncomfortable and difficult grip. The difficulties of providing proper illumination have resulted in the development of many battery or ac-powered illuminated stand magnifiers. As a partial solution to these difficulties, COIL (Berkshire, UK) has introduced a series of tilting stand magnifiers. However, tilting the lens results in distortions and reduces the field of view. A novel design incorporating a prism to tilt the image angle has been reported by Spitzberg *et al.*¹

As with other optical magnifiers, stand magnifiers are also limited by aberrations and vignetting. Typically, their field of view is smaller than other equivalent-power devices.³ The limited field, in most cases, prevents binocular use and thus eliminates the potential benefits of binocular vision. Increasing the lens diameter and the field of view by use of an aspheric design usually increases the optical aberrations as well.

In addition to the problems associated with posture, illumination, and field of view, the actual magnification provided by lens magnifiers is not easily defined or measured by the clinician. The information provided by the manufacturer is frequently inaccurate and misleading,^{4,5} making the prescription of these devices unnecessarily complicated. The fiber-optic reading magnifiers we describe eliminate or substantially reduce all the limitations listed above, thus providing significantly improved stand magnifiers for use by the visually impaired.

In Section 2 of this paper we review the optical characteristics of fiber-optic tapers. Section 3 describes some of the specific requirements of tapers designed to be used as magnifiers. The subsequent sections describe the ergonomic benefits of using the taper as a reading magnifier and detail the benefits of using the modified tapers that

we developed for this application, as compared with existing stand lens magnifiers and with the conventional fiber-optic taper as magnifiers.

2. OPTICAL CHARACTERISTICS OF FIBER-OPTIC TAPERS

Low-vision magnifiers with fiber-optic bundles were proposed by Kantor⁶ and by Pelli *et al.*⁷ These designs, however, used the fiber-optic bundle as an image conduit, and magnification was achieved by means of conventional lens optics. In the case of taper magnifiers, the magnification is obtained with the fibers themselves.

Fiber-optic tapers consist of a large number of optical fibers fused together to form a coherent bundle. The bundle is heat formed, resulting in variation of its diameter from one end to the other. In this process each fiber is stretched and is tapered as well. When carried out under well-controlled conditions, the stretching process produces a taper having a minimum of image distortion. When such a fiber-optic taper is placed with its small face in contact with an object, an enlarged real image appears on the large upper face.

A. Magnification

The magnification of a taper is simply the ratio of the diameters of the end faces. Magnifications in the range of 2–5× are quite practical. When used in the reverse direction the taper functions as a minifier.

B. Numerical Aperture

The numerical aperture (NA) of optical fibers and bundles of fibers is a measure of the angular width of the cone of light that is captured by the fibers. This parameter is measured by the maximum angle of obliquity at which an image is still observable on the face of the bundle (fiber). Beyond this angle the image fades off. For a fiber of uniform diameter (nontapered), the nominal NA is determined by the refractive indices of the materials that constitute the fiber core and cladding. It is given by⁸

$$NA = n_0 \sin \alpha = \sqrt{n_1^2 - n_2^2}, \quad (1)$$

where n_0 is the refractive index of the external medium (for air, $n_0 = 1$); n_1 is the refractive index of the core; and n_2 is the refractive index of the cladding. The angle α is the half-angle of the cone of light captured or emitted by the fiber. This parameter is important in the use of a taper as a magnifier because it determines both the light-gathering capability and the angular field around the taper, from which the enlarged image on the top face of the taper is visible.

In a fiber-optic taper the effective numerical aperture (NA_{Eff}) at the large face is determined by the tapering of the fibers. The reduction in NA at the large end in such fibers is proportional to the magnification. In a cone-shaped fiber (Fig. 1) the following relations hold for the angles α and α' at the two ends:

$$\sin \alpha' = \frac{\sin \alpha}{M}, \quad (2)$$

$$\therefore NA_{\text{Eff}} = \frac{NA}{M}, \quad (3)$$

where M is the magnification and α and α' are the obliquity angles at the small and the large ends, respectively.

In general, it is desirable for NA_{Eff} at the large end of the taper to be as large as possible to provide for both high object illumination and a wide viewing angle. Meeting this goal calls for a high-nominal-fiber NA and also sets limits on the useful magnification. Obtaining a high nominal NA in turn requires the use of a core glass with a high refractive index (which is often a more expensive glass), or a very-low-index cladding glass, or both. Thus there are important trade-offs among NA, magnification, and cost.

C. Viewing Angles and Field of View

The instantaneous field of view of a magnifier is an important parameter.⁹ In the case of a taper, it is simply the angular subtense of the large face of the taper with regard to the observer's eye. When the taper is viewed head on (i.e., normal to the axis), a particularly high NA_{Eff} is not required.

The object field is the width or the diameter of the small end of the taper and is expressed in linear terms, for example, as 1.0 in. (2.54 cm). Thus the desired size of the small bottom face of a taper used as a reading aid is determined by the number of printed letters that one wishes to display simultaneously. Typical newsprint is approximately 15–17 letters/in. (6 letters/cm).

As a magnifier is scanned across a page of text, the scannable field of text that can be read through the magnifier without a change in body posture can be defined. This is small for a typical optical stand magnifier. For the taper it is determined in part by the NA_{Eff} . For example, in tapers with 2.0× magnification and a nominal NA of 1.0, the scannable field easily exceeds the full face of an 8 in. × 11 in. (20 cm × 28 cm) page on a desk, as discussed in detail below. The innovation presented below of tilting the taper toward the observer provides a doubling of even this increased scannable field. The large scannable field is one of the main advantages of the taper over an optical lens magnifier of equivalent magnification.

D. Image Resolution

Image resolution in fiber bundles is generally related to the fiber size. For static resolution (no scanning), a commonly used criterion for resolution is^{7,10}

$$\frac{1}{3d} < R < \frac{1}{2d}, \quad (4)$$

where d is the diameter of the fiber in millimeters and R is the resolution in line pairs/mm.

When one scans the object by moving the fiber bundle, the effective resolution is more than doubled.¹⁰ For per-

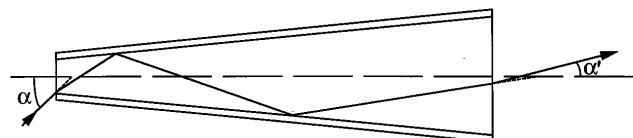


Fig. 1. Effect of the fiber's conical shape on the NA at the large end.

sons with low vision, the image resolution of the magnifier need not be as high as for other applications of tapers,^{7,11} and thus tapers of lower resolution may be used to reduce their cost.

E. Transmission

Each fiber in the taper is composed of a high-index-core glass surrounded by a lower-index cladding. Only the cores transmit imaging light. The light transmission of a fiber-optic taper is given in terms of (1) the internal transmittance of the core glass of the fibers, (2) the Fresnel reflection losses at the faces, and (3) the packing fraction (PF). The PF is the ratio of core area to the total taper face area (i.e., core plus cladding). The thinner the cladding relative to the core, the higher the PF. Thus the transmission through the cores is given by

$$T = PF t_f \exp(-\beta_\lambda L), \quad (5)$$

where t_f is the Fresnel transmission factor, β_λ is the absorption coefficient of the core glass, and L is the length of the taper.

The PF of commonly used tapers is of the order of 70–75%. The claddings of tapers containing extramural absorption (EMA; described in Subsection 2.F) do not transmit light. For tapers without EMA the cladding transmits light for illumination but not for imaging, as explained below. The Fresnel factor is a function of the refractive index of the core glass. For $n_1 = 1.8$, the reflection R_f at a single surface is

$$R_f = 1 - t_f = \frac{(n_1 - 1)^2}{(n_1 + 1)^2} = \frac{0.8^2}{2.8^2} = 0.08, \quad (6)$$

and thus 92% of the light is transmitted. When the taper is used as a magnifier the light makes four surface crossings, reducing the transmitted light by as much as 29% and resulting in a t_f value of approximately 71%. The light transmission can be increased by use of anti-reflection (AR) coatings. The internal transmittance of the core glasses typically used is high (exceeding 98%), except possibly in the blue end of the spectrum, and thus the taper may display a slightly yellow color. Although neutral transmittance is preferred, reduction in short wavelengths is not very detrimental and may be considered an advantage by some patients. (Yellow acetate filters are used over text by some low-vision patients.) At the same time, however, preliminary informal experience with low-vision patients indicates a clear preference for bright and white tapers over even slightly yellow ones.

F. Image Contrast

Loss in contrast transfer through the taper is primarily due to two factors: (1) the percentage of non-image-forming light transmitted by the taper, and (2) any veiling glare due to light reflected from the top face of the taper. The latter is influenced by the ambient light conditions and by the reflectivity of the taper face. This glare can be reduced by means of a low-reflection coating. In general, glare in the form of specular reflections of light sources cannot be avoided from the convex surfaces of lens magnifiers. Nevertheless, such specular reflections can usually be controlled from the flat surface of the taper by

proper positioning relative to ambient light sources or controlled positioning of light sources. Such positioning is made easier by use of the tilted taper configuration described below.

The more important factor in contrast is the non-image-forming light transmitted by the cladding. The cladding material occupies approximately 30% of the area of commonly made high-resolution tapers. This large amount of nonimaging light limits the ratio of contrast transferred through the taper. The lower-resolution requirements of the taper for use by the visually impaired allow the fiber size to be quite large, whereas the absolute cladding thickness can remain small. Thus the PF can be quite high (exceeding 90%), resulting in a concomitant improvement in contrast transfer through the taper (see measurement results in Subsection 5.B below).

If the taper design contains EMA,¹² the stray light through the cladding can be eliminated. The EMA is provided by incorporation of small black glass rods between the clear clad fibers. These absorbing rods, although covering only a small fraction of the cladding area, are able to absorb almost all the stray light in the cladding, leading to a substantial reduction in light scatter in the taper and thus significantly increasing the contrast transfer. At the same time, however, EMA also reduces the light-gathering capability of the taper. Tapers with EMA appear darker than equivalent tapers without EMA. During informal presentation of tapers to visually impaired persons, patients almost uniformly indicated a preference for bright tapers without EMA over those containing EMA. In some cases this preference persisted even though the EMA-containing taper was larger and of higher magnification than the non-EMA-containing taper.

G. Image Distortions

Fiber-optic tapers do not exhibit any of the so-called Seidel aberrations of lenses, such as spherical aberration, chromatic aberration, coma, or astigmatism. When properly made, they exhibit no significant distortion; however, some distortions may result from the various bias angle cuts described below. The methods for controlling or eliminating these distortions are also described.

H. Illumination in Taper Magnifiers

When used as reading magnifiers, tapers also serve to illuminate the object efficiently. This occurs because, under most circumstances, each fiber of the taper serves as a conical light condenser (Fig. 2) and actually increases the

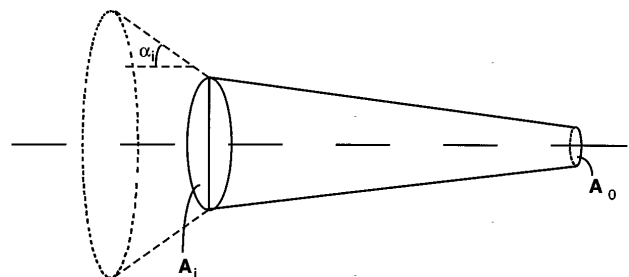


Fig. 2. Schematic conical taper (fiber) illustrating the relations among the face areas at both ends of a tapered fiber and the admittance cone from which light is collected.

illumination of the object over the illumination without the taper. The effect is similar to the light-condensing effect of a lens magnifier, except that for the taper the increased illumination is uniform over the entire field of view.

The light flux incident upon the larger entrance face of the fiber consists of light coming from all the sources above the fibers. However, only that portion which falls within the admittance angle α' can enter the fibers and be transmitted to the object (Fig. 2), where

$$\sin \alpha' = \text{NA}_{\text{Eff}}. \quad (7)$$

Some of the incident light is reflected from the large face, and thus only the light transmitted by the large face is captured by the fibers:

$$\Phi_C = \Phi_i t_f, \quad (8)$$

where Φ_i is the incident flux and t_f is the Fresnel transmission of the large face. This flux reaches the small end of the taper and emerges as

$$\Phi_0 = \Phi_i t_f^2 T_i = T \Phi_i, \quad (9)$$

where T_i is the internal transmission factor and $T = t_f^2 T_i$ is the transmittance of the taper. The illuminance of the large end of the fiber is

$$E_i = \Phi_i / A_i, \quad (10)$$

where A_i is the area of the large end of the fiber. Similarly, the illuminance of the object is

$$E_0 = \frac{T \Phi_i}{A_0}, \quad (11)$$

where A_0 is the area of the small end of the fiber, and

$$A_i = M^2 A_0, \quad (12)$$

where M is the taper magnification. Thus

$$E_0 = E_i T M^2. \quad (13)$$

The multiplying factor of M^2 provides the increased illuminance of the object above that without the taper. This increase in illuminance can be appreciated if the taper is raised slightly above the paper. A bright spot of light can be seen just below the taper.

The luminance perceived when viewing with the taper is simply the illuminance E_0 reduced by the transmittance of the taper (T):

$$L = T E_0 R, \quad (14)$$

where R is the reflectivity of the printed object.

The total flux entering the taper under uniform diffuse illumination is proportional to $(\text{NA}_{\text{Eff}})^2$, and, because

$$\text{NA}_{\text{Eff}} = \text{NA} / M, \quad (15)$$

the total flux entering the taper is proportional to $1/M^2$. This factor appears to cancel exactly the increase in illu-

minance provided by the magnification. Indeed, if the taper is placed under a uniform dome of diffuse light, there will be no increase in object illumination. In fact, illumination will decrease because of the limited transmission. In most environments, however, light sources are of limited angular extent. Thus there is a trade-off, between the admittance cone and magnification, that depends on the way the light sources are distributed. If the light source(s) are favorably distributed or can be controlled to have more of the light included in the admittance cone, a substantial increase in object illumination can be achieved. A higher magnification limits the admittance cone while it increases the object illumination. In general, the higher the nominal NA the better, inasmuch as a higher nominal NA provides a higher NA_{Eff} , even for a large magnification taper. A wider admittance cone makes it easier to capture light sources to increase brightness and also increases the scanning field. Below we show how the effective admittance cone can be increased in a modified taper.

3. OPTICAL REQUIREMENTS OF TAPER MAGNIFIERS

A. Resolution

The ability of the taper to resolve detail depends on the size of the taper fibers that are in contact with the object (usually the small end). If, for example, one wishes to sample print at laser printer quality corresponding to 300 dots/in., then the fiber size should be approximately 80 μm . The fiber size at the large end of the taper is M times as large (where M is the magnification). For a $3.0\times$ taper the fibers at the large end would then be 240 μm . Current tapers of this magnification (used for other purposes) typically have fibers of only 6–20 μm at the large end.

For low-vision observers even the resolution obtained with 240- μm fibers may be unnecessarily high. The magnified image will be, in this case, constructed from approximately 100 dots/in. When viewed from a distance of 10 in. (25 cm) from the large end of the taper, a fiber of 240 μm will subtend approximately 1 mrad (3.4 arcmin), which corresponds to only 3 times the normal acuity limit (roughly 20/70). If the observer can accommodate to shorter distances or uses high-add spectacles to reduce the viewing distance to less than 10 in. (25 cm), the angular subtense of the fiber increases accordingly, but the fiber is still unlikely to be resolved by the low-vision observer. Furthermore, it has been shown that the associated sampling rates (12×12 samples/letter), even when resolved, do not reduce reading rates.¹³ In fact, for low-vision patients a sampling rate as low as 4×4 samples/letter does not reduce reading rates in any meaningful way.

Two phenomena increase the effective resolution of a taper magnifier further: naturally occurring gray-scale antialiasing, and dynamic resolution. It has been shown that a low-resolution magnified image such as that obtained by pixel replication on computer or low-vision closed-circuit TV systems reduces reading speeds.¹⁴ Antialiasing effects obtained either through the use of optical lens magnification in closed-circuit TV systems or by means of gray scale in computerized text displays

increase the reading speed of normal¹⁵ and visually impaired readers.¹⁴ The tapers provide such gray-scale antialiasing effects when they are used as reading magnifiers. Fibers at the small end of the taper falling on the border of a letter will present a gray image pixel at the large end corresponding to the average reflectance across a given small end fiber. Thus the letters sampled by a low-resolution taper will have a smoother, more natural appearance than those used by Legge *et al.*¹³ Furthermore, as mentioned above, Kapany *et al.*¹⁰ demonstrated that the scanning of a fiber-optic bundle across the target may increase the effective resolution by as much as 100%. This benefit will be afforded to those users of tapers who have impaired or normal vision.

B. Size of the Taper Magnifier

The size of the taper is determined by the width of the object to be magnified and by the magnification. Typical newsprint contains approximately 15–17 letters/in. Whereas the minimum number of letters required for reading at a maximum reading rate with a magnifier is not known, for computer scrolled or scanned text, only four letters are needed.¹¹ For stand magnifiers many more letters may be needed for optimal reading, presumably because of the help larger fields afford the reader in text navigation (i.e., finding the next line).^{3,16,17} Yet Beckman¹⁸ demonstrated that only four letters are necessary for maximum reading rate in free reading on a closed-circuit TV. Thus field diameters in the range of 15–25 mm are expected to be appropriate (so as to cover from 10 to 17 letters simultaneously; smaller fields may prove sufficient). Encompassing the range of magnifications that may prove optimal for low-vision observers, namely, from 2.0 \times to 5.0 \times , will require the use of tapers with a large-end diameter of as much as approximately 3 in. (75 mm), although a diameter of 2.5 in. (63 mm) may prove sufficient and would help limit the cost and the weight.

C. Contrast

The contrast transfer through the taper should be as high as possible.¹⁷ Below we describe the increase in contrast achievable with a reduced cladding-to-core fraction and with the use of AR coating on either or both faces of the taper.

D. Scanning Field

A magnifier that requires minimal body movement while the user scans a page of text would be of great utility for many elderly low-vision users. We know of no published data on the actual requirements of and preferences involved in this task. Below we demonstrate that the modified tapers we have developed have more than double the scanning range of comparable-power lens magnifiers.

4. TAPER MODIFICATION FOR BETTER ERGONOMICS

The taper provides actual-size magnification with an edge-to-edge full field of distortion-free imaging that may be viewed comfortably with both eyes or with the aid of reading spectacles for added magnification and clarity.

Holding the taper and manual control are easier because there is no need to allow for illumination from the side. Moreover, one can ease the postural requirements by tilting the taper's upper face toward the user. Tilting the upper face toward the user also increases the flexibility of controlling the illumination and helps to avoid disturbing specular reflections. Most importantly, the tilting of the taper increases the scanning range.

A. Increasing the Scanning Field of Taper Magnifiers

The scanning field is defined as the distance that the magnifier can be moved and still continue to display a bright, sharp image while the subject's head is fixed in one position (head or eye tilt is permitted). One can increase the scanning field by slanting the taper to tilt the image toward the observer. Such slanting can be achieved in a variety of ways. The simplest method is to tilt the taper toward the users by cutting the bottom, smaller face of the taper on a bias (Fig. 3).

The scanning range L can be calculated for such a tilted taper from the parameters shown in Fig. 4, where D is the closest distance from which an image will be visible on the face of the taper (D can be zero or even negative) and $D + L$ is the farthest distance from the observer at which the image is still visible. As illustrated, these distances are determined by the obliquity angle α' and by the height of the observer's eye. Thus

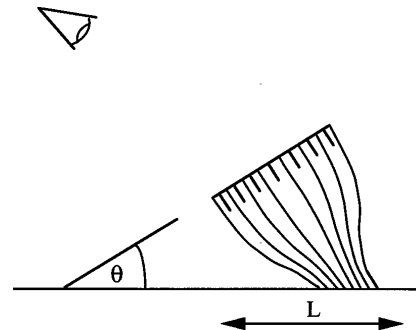


Fig. 3. Taper tilted toward the user by cutting the smaller face on a bias.

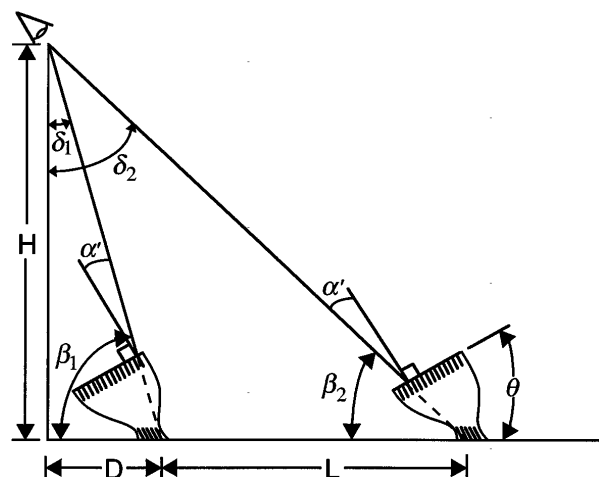


Fig. 4. Parameters associated with the calculations of the expanded scanning field of a taper with a bias cut at the small end.

Table 1. Effective Scanning Range ($L+D$) Calculated for a Number of Tapers^a

Magnification (M)	Admittance (α')	Tilt (θ)	L	D	$(L + D)$
2.0×	30°	0°	11.55 (29.4)	-5.77 (-14.7)	5.77 (14.7)
2.0×	30°	30°	17.3 (43.9)	0 (0)	17.3 (43.9)
2.0×	30°	13°	12.4 (31.5)	-3.1 (-7.9)	9.33 (23.7)
2.0×	30°	36°	21.41 (54.4)	1.05 (2.7)	22.46 (57.1)
2.75×	21.3°	0°	7.81 (19.8)	-3.90 (-9.9)	3.90 (9.9)
2.75×	21.3°	21.3°	9.20 (23.4)	0 (0)	9.20 (23.4)
2.75×	21.3°	15°	8.46 (21.5)	-1.11 (-2.8)	8.46 (21.5)
2.75×	21.3°	25°	7.81 (19.8)	0.64 (1.6)	10.47 (26.6)
3.0×	19.5°	0°	7.07 (18.0)	-3.54 (-9.0)	3.54 (9.0)
3.0×	19.5°	19.5°	8.08 (20.5)	0 (0)	8.08 (20.5)
3.0×	19.5°	12°	7.43 (18.9)	-1.31 (-3.3)	6.12 (15.5)
3.0×	19.5°	25°	8.85 (22.5)	0.97 (2.5)	9.82 (24.9)
4.0×	14.5°	7°	5.25 (13.3)	-1.31 (-3.3)	3.93 (9.9)

^aIn all the cases the height of the eye above the object, H , was 10 in. (25.4 cm), with a nominal NA of 1.0. The boldface entries are for the tapers measured (Table 2). L , D , and $(L + D)$ measurements are given in inches and, parenthetically, in centimeters.

$$\tan \delta_1 = \tan(\theta - \alpha') = D/H, \quad (16)$$

$$\tan \delta_2 = \tan(\theta + \alpha') = (D + L)/H, \quad (17)$$

$$\therefore L = H[\tan(\theta + \alpha') - \tan(\theta - \alpha')]. \quad (18)$$

In the nominal case, where $D = 0$ (i.e., with the taper being scanned from just under the eyes and away from the observer), the optimal tilt angle is $\theta = \alpha'$. Smaller angles ($\theta < \alpha'$) will result in a loss of scanning range ($D < 0$), whereas larger angles of tilt are possible but will move the scanning range away from the user ($D > 0$). Table 1 presents the calculated scanning field for a number of tapers.

The lateral range (right to left) is also limited by the total effective range. By slightly rotating the slanted taper (toward the user) while scanning laterally, one can maximize the lateral range. The full scanning field covered in this way will be bound by a circle of radius equal to the scanning range ($D + L$) centered at the observer. Thus, for most of the forward field (to as much as 85%), a lateral field wider than the scanning range ($D + L$) can be covered. The calculated effect of tilting the taper on the scanning range is shown in Table 1.

B. Controlling the Illumination

In addition to providing an increased scanning range, tilted tapers also provide better control of illumination. User control is improved for both collecting the ambient light and avoiding the glare resulting from specular reflections of any bright light sources from the upper face of the taper. Because of the tilt of the upper face, rotation of the taper around a vertical axis allows the user to include within the taper's admittance cone a light source (a window or a ceiling light fixture) that would otherwise lie outside the admittance cone of the same taper without the tilt (Fig. 5). Once a source lies within the admittance cone, its light is concentrated by the taper and results in a brighter image. Specular reflections from bright sources that would be reflected from the polished upper surface of the taper can also be redirected by the same slight rotation of the taper away from the user's eyes. Such re-

flections are almost impossible to avoid from the curved surface(s) of a lens magnifier.

5. MEASURED CHARACTERISTICS OF TAPERS

A. Scanning Range of Taper Magnifiers versus Conventional Stand Magnifiers

The scanning ranges of standard and tilted tapers were measured and were compared with the scanning ranges of commercially available, commonly used lens stand magnifiers. We evaluated three types of tapers: the standard high-resolution tapers made with no EMA (Galileo Electro Optics, Sturbridge, Mass.; magnification, 2.0×); those made with EMA (Schott Fiber Optics, Southbridge, Mass.; magnification, 2.75×); and those that we designed for low-vision use, with lower resolution and thin cladding (magnification, 3.0× and 4.0×). These tapers had equivalent viewing distances (EVD's) of 5.0, 3.6, 3.3, and 2.5 in. (12.5, 9.1, 8.3, and 6.3 cm, respectively) for the 2.0×, 2.75×, 3.0×, and 4.0× tapers, respectively, and for an eye-to-lens distance of 10 in. (25.4 cm).¹⁹ The EVD is a measure of the effective magnification in terms of retinal image size. It specifies the observation distance required

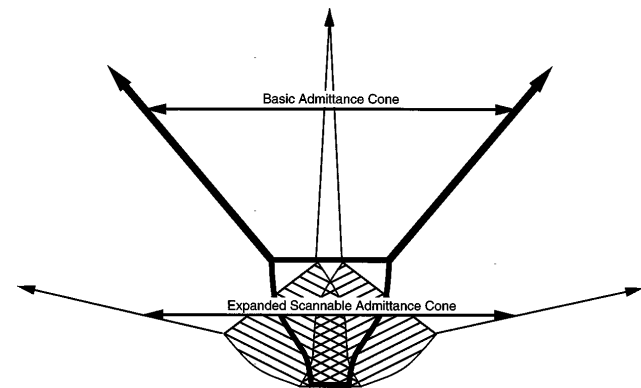


Fig. 5. Increase in effective admittance cone provided by scanning of a tilted taper (thin lines) as compared with the basic admittance cone of a simple upright taper (thick lines).

for obtaining the same retinal image size without the use of the magnifier.¹⁹ The tapers were compared with popular lens stand magnifiers with comparable EVD's:

- (1) Eschenbach 2627; marked 4.0×; measured 13.3*D*; enlargement ratio (ER), 3.6×; EVD, 4.8 in. (12.3 cm);
- (2) COIL 5214 tilted; marked 4.0×; measured 10.8*D*; ER, 8.5×; EVD, 4.4 in. (11.1 cm);
- (3) Selsi 402 Jupiter; marked 4.0×; measured 14.3*D*; ER, 3.7×; EVD, 4.7 in. (11.9 cm);
- (4) COIL 6279 (illuminated); marked 5.4×; measured 18.2*D*; ER, 7.1×; EVD, 3.2 in. (8.2 cm);
- (5) COIL 5226; marked 6.0×; measured 23.7*D*; ER, 10.8×; EVD, 2.4 in. (6.1 cm);
- (6) Peak 2018; marked 8.0×; measured 26.0*D*; ER, 8.9×; EVD, 2.4 in. (6.2 cm).

Note that, although the first three stand magnifiers are rated as 4×, they provide a retinal image size (determined by the EVD) similar to that provided by a taper of approximately 2.0×, magnification with an eye-to-lens distance of 10 in. (25 cm). Similarly, the next stand magnifier has an EVD similar to the 3.0× taper, and the last two have EVD values close to that of the 4.0× taper. These relations illustrate the fact that the effective magnification provided by tapers is much larger than that provided by similarly rated stand magnifiers. For presbyopic users the tapers used at this distance require a +4.00*D* reading add, whereas most stand magnifiers can be used with a +2.50 add. If the eye-to-lens distance is reduced to 4 in. (10 cm), the advantage of the taper increases even further. At that shorter distance, and with the appropriate reading add, the 3.0× taper provides an effective magnification similar to that of stand magnifiers rated as 15× (i.e., COIL 4215 or Peak 2023).

Three subjects, with vision corrected to 20/20 by use of spectacles or contact lenses, determined the limit of the scanning range for all the magnifiers. For each subject, the head was placed in a chin rest adjusted to bring the

height of the lateral cantus of the eye to 10 in. (25 cm) above the reading document. The document contained one long column of continuous text printed by a laser printer, with a Times Roman 10-point font equivalent to newspaper print (approximately 1*M* size). The subjects moved the magnifier across the column of text and read the text through the magnifier. They indicated to the experimenter the position of the magnifier corresponding to the farthest usable position. This position was defined for the case of the tapers as the position in which text at the far edge of the taper could not be read or seen while text slightly closer to the nearer end of the taper was still readable. For the lens magnifiers the end of the range was defined as the position of the magnifier in which a full line of text (9 cm wide) could be read despite significant distortions. Note that for both the tapers and the lens magnifiers the quality of the text seen through the lens at the end of the range was marginal. In both cases, contrast was substantially reduced, and for the lens magnifiers the text was highly distorted.

The resulting ranges determined by the three observers are included in Tables 2 and 3. A comparison with Table 1 shows that the measured scanning ranges of the tapers agree well with their calculated parameters. The advantage of the tilted tapers over the lens magnifiers is apparent (compare Tables 2 and 3). The range of the tilted 2.0× taper (boldface entry in Table 2) is almost double that of the equivalent lens magnifiers (boldface entries in Table 3), even though a bias cut of less than half the optimal angle was used. Similarly, compare the range of the tilted 3.0× taper (italic entry in Table 2) with those of the equivalent stand magnifiers (italic entry in Table 3), and compare the range of the tilted 4.0× taper (underscored entry in Table 2) with those of the corresponding entries in Table 3. The actual benefit of the taper is even larger than that indicated by the numerical results because the image through the lens magnifiers becomes highly distorted even at distances much shorter than the scanning range defined here.

Table 2. Measured Scanning Fields of Tapers^a

Magnification (<i>M</i>)	Admittance (α')	Tilt (θ)	EVD [at 10 in. (25.4 cm)]	Measured Range	Calculated (<i>L</i> + <i>D</i>)
2.0×	30.0°	0°	5.0 (12.7)	6.77 ± 0.59 (17.2 ± 1.5)	5.77 (14.7)
2.0×	30.0°	13°	5.0 (12.7)	9.83 ± 0.68 (25 ± 1.7)	9.33 (23.7)
2.75×	21.3°	0°	3.6 (9.2)	4.25 ± 0.25 (10.8 ± 0.6)	3.90 (9.9)
2.75×	21.3°	15°	3.6 (9.2)	9.83 ± 0.63 (25 ± 1.6)	8.46 (21.5)
3.0×	19.5°	0°	3.3 (8.4)	3.28 ± 0.25 (8.3 ± 0.6)	3.54 (9.0)
3.0×	19.5°	12°	3.3 (8.4)	6.08 ± 0.22 (15.4 ± 0.6)	6.12 (15.5)
4.0×	14.5°	7°	2.5 (6.4)	<u>5.71 ± 0.80 (14.5 ± 2.0)</u>	3.93 (10.0)

^aIn all the cases nominal NA was 1.0, and *H* = 10 in. (25.4 cm). See Subsection 5.A for explanation of boldface, italic, and underscored numbers.

Table 3. Measured Scanning Fields of Lens Stand Magnifiers^a

Type	Magnification (Marked)	EVD [at 10 in. (25.4 cm)]	Measured Range
Eschenbach 2627	4.0×	5.2 (13.2)	4.50 ± 0.50 (11.4 ± 1.3)
COIL 5214 (tilted)	4.0×	4.8 (12.2)	5.75 ± 0.43 (14.6 ± 2.0)
Selsi 402 Jupiter	4.0×	4.4 (11.2)	4.33 ± 0.29 (11.0 ± 0.7)
COIL 6279 (illuminated)	5.4×	3.2 (8.1)	4.29 ± 0.56 (10.9 ± 1.4)
COIL 5226	6.0×	2.4 (6.1)	<u>4.38 ± 0.94 (11.1 ± 2.4)</u>
Peak 2018	8.0×	2.4 (6.1)	<u>2.83 ± 1.01 (7.2 ± 2.6)</u>

^aIn all the cases *H* = 10 in. (25.4 cm). See Subsection 5.A for explanation of boldface, italic, and underscored numbers.

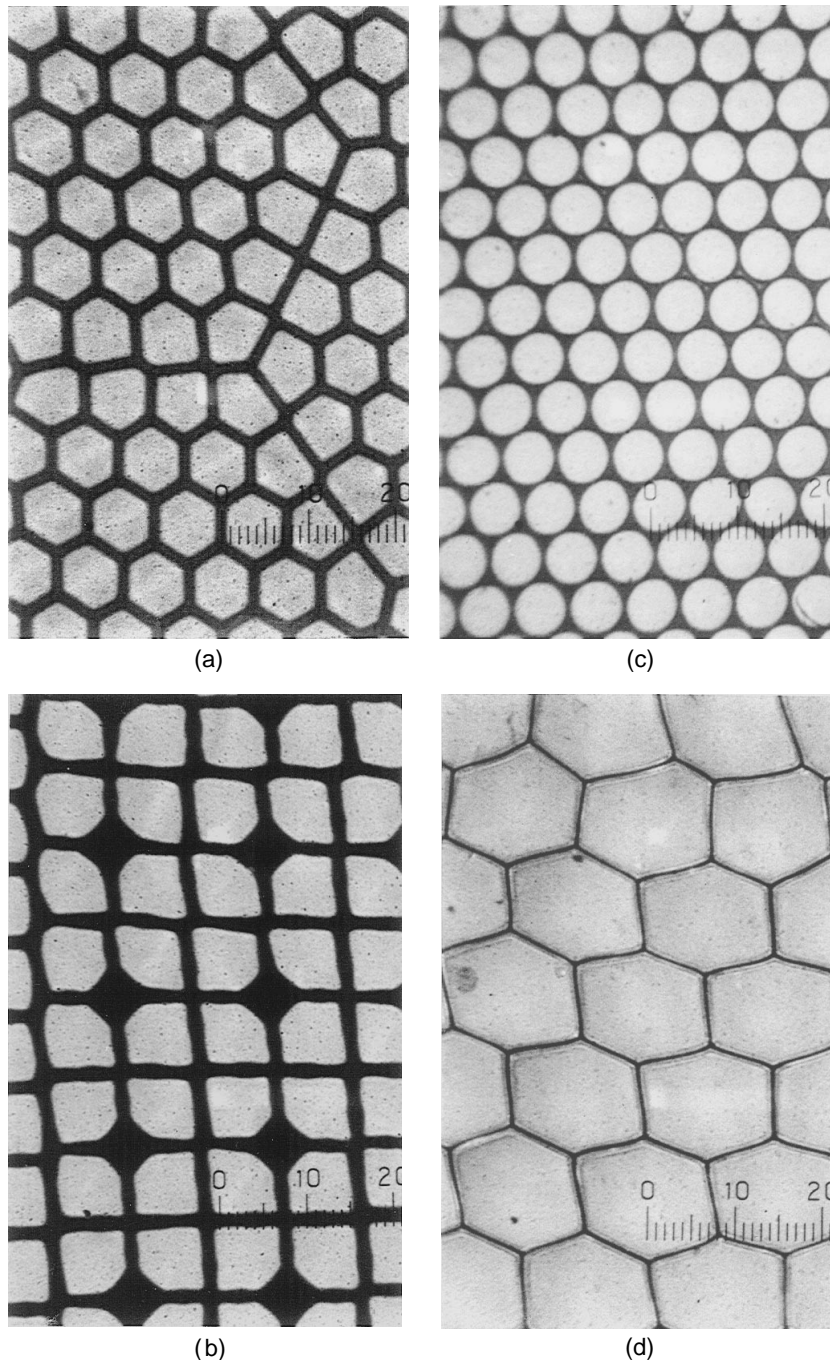


Fig. 6. Microscopic images of the different fiber-optic bundles used. Note that the magnification used in photographing each was different. (a) Galileo material with fibers of $20\ \mu\text{m}$ and 30% cladding. (b) Schott EMA material with fibers of $8\ \mu\text{m}$ and 25% cladding. (c) SBIR 1 material designed for reading magnifiers with core diameters of $300\ \mu\text{m}$ and 17% cladding. (d) SBIR 2 material with fibers of $300\ \mu\text{m}$ and 7% cladding.

B. Contrast Transfer Ratio Through Tapers

To evaluate the contrast transmittance of tapers used as magnifiers, we measured the contrast of a square-wave grating through five different types of fiber-optic bundle materials used in the tapers. The material, a parallel (nontapered) bundle slab of approximately 2 in. (5 cm), was cut from the large ends of tapers of various types used in the scanning range measurements described above. Parallel slabs were used to avoid the confounding effects of luminance changes associated with the taper magnification. The Schott EMA material had square fibers of $8\ \mu\text{m}$ and a clad fraction of 25%, including the EMA.

The Galileo material had hexagonal fibers of $20\ \mu\text{m}$, with a clad fraction of 30%. The materials designated SBIR 1a and 1b, designed by us in this investigation specifically for use as reading magnifiers for the visually impaired, were made from the same material, with a clad fraction of 17% and round fibers in hexagonal packing of 240 and $300\ \mu\text{m}$, respectively. The SBIR 2 material had hexagonal fibers of $300\ \mu\text{m}$ and a clad fraction of 7% (Fig. 6). (SBIR stands for Small Business Innovative Research.)

The luminance values of the bright and the dark stripes (7 mm/stripe) of the gratings printed with a laser printer

on white paper were measured with a Minolta LS100 luminance meter (1° test spot). The minimum and the maximum luminances were then measured through the various tapers placed over the printed gratings. Measurements were taken in a dimly lit room with an adjustable arm lamp (60-W incandescent bulb) positioned above the tapers. The luminance meter was positioned with a tripod approximately 20° off the normal to the taper surface. The light fixture was shifted very slightly off the normal to the tapers toward the meter to direct specular reflection of the bulb away from the meter. This arrangement represents optimal conditions for proper use of the light when using a taper as a reading magnifier.

The Michelson contrast was found to be 0.83 for the paper target alone. The luminance values measured through the various tapers and the contrasts calculated from them are given in Table 4. The contrast transfer was calculated as the ratio of the contrast through the fiber-optic bundles to the contrast measured without the bundles (0.83).

As expected, the material with the EMA provides much better contrast transfer than the standard material without EMA. The increase in contrast comes with the cost of a substantial reduction in brightness. The contrast reduction of the standard (non-EMA) material results from nonimaging light being scattered in the cladding of the fibers. In the typical tapers used here, the cladding occupies 25–30% of the taper's face area. The tapers we designed for use as low-vision magnifiers have larger cores and a much smaller cladding-to-core ratio to improve contrast transfer. The contrast transfer increases that we measured are consistent with the increases in the core-to-cladding ratio implemented. Other factors, such as fiber diameter, may have an effect as well, as is evident from the difference in contrast transfer through tapers marked SBIR 1a and 1b. These two materials had identical core fractions (made from the same batch of multifibers) and differ only in the fiber diameter.

Next we discuss the effect of AR coating. The high index of the core glass results in substantial Fresnel reflection at the glass-air interface. For $n = 1.8$, the reflectance is approximately 8% at each surface. In the taper magnifier these reflections occur four times, so that the total loss of light is as much as 29%. It appears, therefore, that AR coating used over both of the taper sur-

faces could substantially increase the contrast because the reflected light serves as glare light and reduces the print contrast.

AR coating was applied to a standard (non-EMA) commercial taper (Galileo) by means of a quarter-wavelength layer of MgF_2 deposited on both surfaces (American Optical Co., Southbridge, Mass.). On each surface only half the surface was coated, and the lines separating the two surfaces were orthogonal. This arrangement allowed us to evaluate and to compare four different levels of coatings on the same taper. The contrast transfer ratio was measured as described above. The results are given in Table 5.

Contrast transfer increased more with AR coating when the bottom face was coated than when the top face was coated. Coating of both faces increased the contrast transfer substantially, from 0.67 to 0.87. The use of coating on the bottom surface should be further investigated because the continuous friction of the taper against the reading materials may result in stripping of the coating even if a hard, scratch-resistant coating is used. Furthermore, with the increase in contrast afforded by the increase in core-to-cladding ratio, the additional benefit provided by the AR coating diminishes.

6. BIAS-CUT DISTORTIONS

A. Anamorphic Distortion

Cutting the smaller face of the taper at an angle (Fig. 3) will introduce a small amount of anamorphic magnification into the image displayed on the large face. That is, the magnification will be different for the vertical and the horizontal meridians (diameters) of the transmitted image. The amount of this difference (more correctly, the ratio of these magnifications) is simply the cosine of the angle of cut relative to the normal (perpendicular) face (Fig. 3, angle θ). For a relatively large angle cut of $\theta = 30^\circ$, which is the optimal angle for a $2.0\times$ taper (see Table 1), $\cos 30^\circ = 0.87$. For a more typical value of $\theta = 15^\circ$ ($\cos 15^\circ = 0.97$), the distortion is only 3%, which is scarcely noticeable. This anamorphic magnification causes the vertical magnification of the scanned text to be slightly less than the horizontal, so that enlarged objects appear slightly wider (or less high) than normal when viewed with the tilted taper. Even this

Table 4. Measured Contrast Transfer of Bundles of Various Materials^a

Taper Type	L_{\max} (cd/m ²)	L_{\min} (cd/m ²)	Contrast	Contrast Transfer
Target (no taper)	108	9.9	0.83	NA
EMA (Schott)	52	9.9	0.68	0.82
Standard (Galileo)	107	26.8	0.60	0.72
SBIR 1a	90	15.6	0.70	0.85
SBIR 1b	96	20	0.66	0.79
SBIR 2	100	12.4	0.78	0.94

^aLuminance was measured with a square-wave target of 14 mm/cycle.

Table 5. Effect of Antireflection Coating on Contrast Transfer of Standard Taper

Surface Coated	L_{\max} (cd/m ²)	L_{\min} (cd/m ²)	Contrast	Contrast Transfer
No coating	198	56.1	0.56	0.67
Top face	243	65.9	0.57	0.69
Bottom face	190	34.4	0.69	0.83
Both faces	224	35.6	0.73	0.87

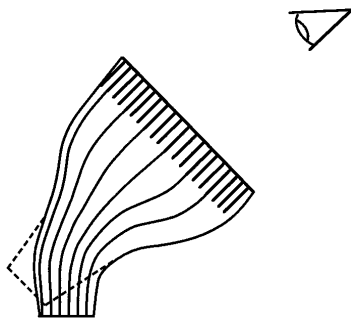


Fig. 7. Combination of tapering and bending that can lead to a tilted taper, as shown here, that is free of distortion and provides the same benefits in scannable range and light control as the taper shown in Fig. 3.

small difference can be reduced or eliminated in a number of ways, as shown below.

B. Keystone Distortion

If the cut through the small end of the magnifier is made through the tapered portion of the magnifier, an additional type of distortion will result, namely, keystone distortion. When cut through the tapered portion, different regions of the upper taper face provide different magnifications. The region closer to the user provides less magnification than the region farther away from the user. The level of distortion depends on both the angle of cut and the steepness of the tapering at the level of the cut. Thus, if a rectangular grid is examined through the taper, it will appear with this keystone distortion. At the same time, however, because this distortion is small, it is not immediately apparent when the taper is used for reading, inasmuch as each line of text has a uniform magnification and the difference between the magnifications of adjacent lines is small and unimportant. The distortion is easy to note when the taper is placed perpendicular to the line of text such that the change of magnification occurs along one line of text.

C. Reducing the Bias-Cut Distortions

1. Bending the Taper

One method for eliminating all anamorphic magnification and keystone-like distortion is to provide the tilt by using a bend of the smaller end of the taper, as shown in Fig. 7, instead of the bias cut. The combination of tapering and bending of a fused bundle of optical fibers may be achieved in either a single step or a combination of forming steps. We perform the usual tapering process by heating the central region of a cylindrical bundle or boule of fused optical fibers and pulling the heat-softened boule into an hourglass shape. We can then achieve bending of the dual taper by tilting one end relative to the other, either as part of the same heating cycle or as a separate operation. In either case, two essentially identical tapers can be made from the single boule. In use, either taper will be tilted toward the user to provide the increased scanning range and will be free from the image distortions of the tapers tilted by a bias cut of the bottom end. We are not aware of any tapers manufactured in this way.

2. Bias Cut at the Large End

A second method for reducing or eliminating the anamorphic and keystone-like distortion of the taper bias cut at the smaller end involves a second, almost parallel, cut

of the taper at the large end (Fig. 8). This cut creates the opposite distortion and thus can be used to cancel the distortions created by the small-face bias cut. The second cut can eliminate the anamorphic distortion and can reduce the keystone for all taper designs. Moreover, elimination of all the distortions is possible in some designs, as discussed in Appendix A.

Although the bias cut at the large end is made away from the user, it still directs the light toward the user. This is due to the prismatic effect at the tip of each fiber. With the upper face cut, the light emerging at the center of each fiber passes through a glass-air interface at an angle. This refraction causes emerging rays to refract toward the user (Fig. 8). The optimal deviation or tilt of the axial ray from the vertical is slightly more than the admittance half angle α' determined by the NA. With this level of tilt the same condition of slightly positive range D , described above, will be achieved. The tilt of the axial ray due to the bias cut of the top face is nearly twice the cut angle. For core glass with index n , a cut at angle γ will result in the axial ray in the air shifted at angle γ' to the normal n , where

$$\sin \gamma' = n \sin \gamma. \quad (19)$$

For $n = 1.8$, a combined parallel cut of both surfaces of the taper at angle γ will result in an axial ray tilt of approximately 2γ from the vertical. The fibers will arrive at the top at an angle of γ , and the top face cut will shift the ray approximately 2γ from the normal, which, because of the parallel cut at the top face, is oriented vertically again (see Fig. 8). Thus one can achieve optimal tilt by cutting both the bottom and the top faces at angles $\gamma = \alpha/2$. This smaller angle of cut reduces both the anamorphic magnification effect and the keystone distortions to negligible levels. Further reductions in the distortions can be achieved by proper design of the angle of cut at the top face. Such designs are discussed further in Appendix A.

In addition to reducing distortions (and thus also preserving magnification), the cut at the top of the taper, when combined with the nearly parallel cut at the bottom, permits better control of light collection and specular reflections. With this design the taper can be used effectively, with illumination coming over the user's shoulder onto the reading material (Fig. 9). This illumination arrangement is recommended for the visually impaired and

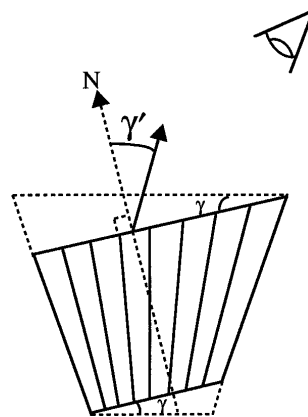


Fig. 8. Schematic illustration of a taper with a bias cut at the bottom end combined with a parallel cut at the top end. The refraction of the light at the top face results in a net tilting toward the observer.

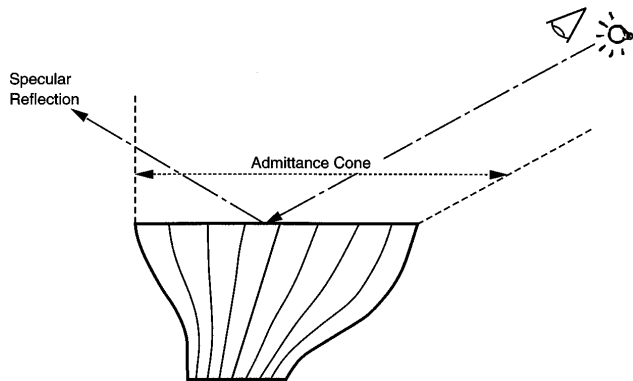


Fig. 9. Schematic illustration of a taper with both ends cut on a bias. This configuration permits control of specular reflections when the light source is behind the reader, as well as complete elimination of specular reflections while maintaining the expanded scanning range of the tilted taper with reduced or no distortions.

for comfortable reading, in general, to prevent glare from the source from reaching the user's eyes. A taper cut as described above will have a flat top surface parallel to the desk, and specular reflections from light sources behind the user will be reflected away from the user's eyes (Fig. 9).

Although the top surface of the taper is horizontal, its admittance cone is tilted relative to the vertical axis. Thus the position of the admittance cone in space can be controlled by slight rotation of the taper, as with the tilted top taper, to scan the environment for better collection of light from ambient sources, as described above.

7. DISCUSSION

The outstanding performance of even existing tapers as reading magnifiers is self-evident, yet they have not found their way into low-vision rehabilitation services. The modifications described here, including the bias cut and the increased contrast by reduction in clad thickness, would make them even more appropriate. We believe that the main obstacle to wider use has been the high cost. Therefore we are working to produce lower-resolution tapers of useful magnification, using materials and production designs that will bring the price into a manageable range.

The increase in the fiber size allows two or more steps to be eliminated from the manufacturing process. With the simplification of the manufacturing process and the use of lower-cost glass for the fibers, the manufacturing cost can be reduced sufficiently to make the device affordable for the many visually impaired persons that could benefit from it.

APPENDIX A

This appendix addresses the optimal angle of cut, made at the top face of a taper, that is needed to minimize or to eliminate the distortions resulting from cutting the bottom face at an angle θ . Two different cases are analyzed below.

1. Cutting across the Tapered Portion at the Top Face

We first analyze the ideal case of a radial (conical) taper (Fig. 10). If, before the bias cut at the bottom face, the taper had a magnification $M = T/B$, then from similar

triangles we can see that

$$M = T/B = R/(R - H). \tag{A1}$$

The anamorphic magnification will be such that the ratio of magnifications in the two main meridians will be B_s/B . We can calculate this ratio from the following geometric relations shown in Fig. 10:

$$\frac{\sin 2\omega}{B_s} = \frac{\sin \beta}{S_1} = \frac{\sin \beta \cos \omega}{R - H}. \tag{A2}$$

Because

$$S_1 = \frac{R - H}{\cos \omega}, \tag{A3}$$

$$\beta = (\pi/2) - \omega - \theta, \tag{A4}$$

then

$$\frac{\sin 2\omega}{B_s} = \frac{\sin[(\pi/2) - \omega - \theta] \cos \omega}{R - H}, \tag{A5}$$

$$R - H = \frac{B}{2 \tan \omega}, \tag{A6}$$

and thus

$$\frac{B_s}{B} = \frac{\sin 2\omega}{\sin[(\pi/2) - \omega - \theta] \cos \omega 2 \tan \omega} = \frac{\cos \omega}{\cos(\omega + \theta)}. \tag{A7}$$

Note that, because B_s/B is independent of H , the same formulation will apply to T_s/T , and therefore the anamorphic distortions are corrected with parallel cuts.

2. Blended Taper: Cutting the Top at the Parallel Portion

Most useful tapers are blended into a parallel bundle at the top. In this case the top face is cut at an angle ϕ

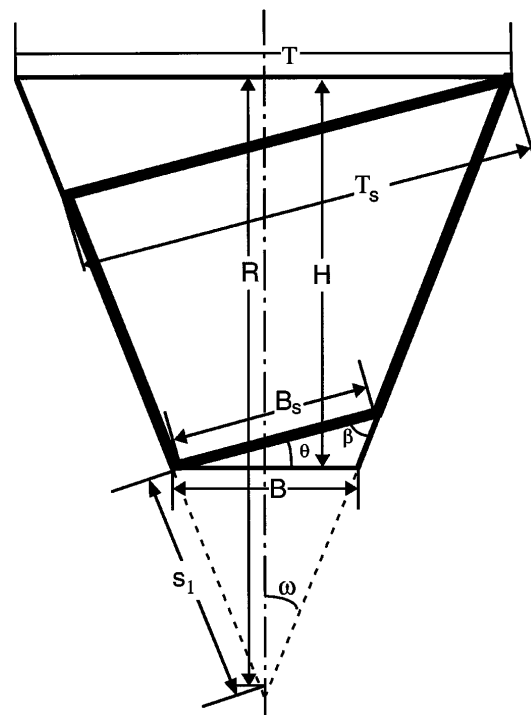


Fig. 10. Parameters associated with a conical taper cut on a bias on both faces through the tapered portion.

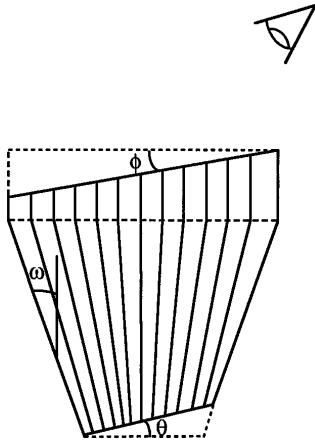


Fig. 11. Geometrical relations for a blended taper whose top face is cut through the parallel, nontapered portion.

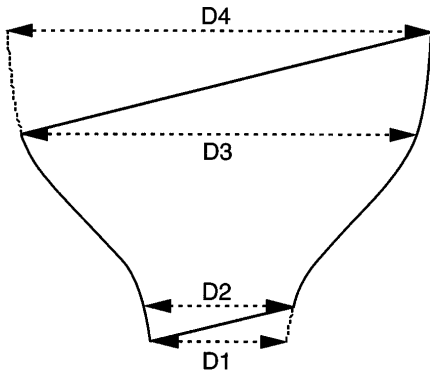


Fig. 12. Geometrical relations required for making the large-face cut to correct the keystone distortion due to the small-face bias cut of a taper of general shape.

through the top, parallel portion of the taper (Fig. 11), and, when we consider the parameters of Fig. 10,

$$T_s = T \sec \phi = \frac{T}{\cos \phi}, \quad (\text{A8})$$

$$\frac{T_s}{T} = \frac{1}{\cos \phi}. \quad (\text{A9})$$

For anamorphic-magnification correction we require that

$$B_s/B = T_s/T. \quad (\text{A10})$$

Because B/B_s is the same as in Eq. (A7) above,

$$\cos \phi = \frac{\sin[(\pi/2) - \omega - \theta]}{\cos^2 \omega} = \frac{\cos(\omega + \theta)}{\cos^2 \omega} \quad (\text{A11})$$

is required for full correction of the anamorphic magnification. For example,

$$\text{if } \omega = 20^\circ, \theta = 20^\circ, \text{ then } \phi \approx 30^\circ.$$

For case 2, we cannot correct the keystone distortion at all by cutting through the parallel portion. For case 1, the parallel-cut solution will correct this problem as well, as can be shown by use of similar triangles.

For the more typical taper shape, which is not an ideal

cone, an optimal solution for the elimination of the keystone distortion can be found with the relationship illustrated in Fig. 12:

$$D_1/D_2 = D_3/D_4. \quad (\text{A12})$$

In any case, a nearly parallel bias cut at the large face through the tapered portion of the taper will reduce the distortions to a negligible level while maintaining all the advantages of the tilted taper in controlling illumination and increasing scanning range.

ACKNOWLEDGMENTS

This research was supported in part by National Institutes of Health grants (EY10500 to W. Siegmund and EY05957 and EY10285 to E. Peli).

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